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A MATHEMATICAL MODEL OF THE PHENOMENON OF RADIOACTIVE FALLOUT

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To describe the phenomenon of fallout we must, of course, begin with the nuclear detonation. The main distinguishing feature of a detonation of this type is a tremendous release of energy almost instantaneously and within a very small space. The actual amount of energy in calories is related to kilotons of yield in the ratio 10^{12} to 1 according to the Atomic Energy Commission book Effects of Atomic Weapons. Or, in other words, one kiloton is equivalent to one trillion calories of energy.

To get a meteorological comparison for this amount of energy, we might compute the energy released by the condensation of water. Suppose we consider a depth of rainfall of one centimeter over one square kilometer, or, 10^{10} gms of water. Allowing 540 calories per gram of water condensed, the energy released would be 5.4×10^{12} calories, or approximately 5 kilotons. The big difference, of course, is the fact that the rainfall releases its energy throughout a large volume of atmosphere and over a period of time measured in minutes or hours, whereas the bomb releases its energy within a very limited space and in a matter of seconds.

This release of energy creates a hot bubble of air which rises because of its buoyancy. The characteristic mushroom-shaped cloud associated with nuclear explosions is formed by water vapor condensed as a result of vertical lifting. The temperature gradient between the hot bubble and the surrounding atmosphere causes a strong toroidal circulation.

Another unique feature of a nuclear explosion is the formation of fission fragments. These are the end products of the splitting of the

Uranium atoms and are almost all radioactive. Fission fragments, together with any other debris mixed into the fireball at an early stage, are carried aloft, and as the cloud rises, some of the material may be left behind forming a wake or stem. The cloud will stop rising when the energy available to lift it has dissipated. At this time, which is referred to as the time of stabilization, the spatial distribution of the radioactivity provides one set of the initial conditions required in the fallout model.

Within this stabilized cloud are vitually all the fission products formed by the nuclear detonation. Radioactive particles may be formed by condensation of vaporized material or by the coalescence of fission products with solid or liquid earth particles. The size of the particles is determined largely by the initial conditions of the burst; if little solid material is taken into the cloud, the particles will in all likelihood be small; if, on the other hand, the device is detonated on the surface, many tons of earth will be swept into the cloud and a sizeable fraction of the fission products will be contained in large particles. The size of the particles containing radioactive elements has an important bearing on the fallout problem. The radioactivity located in or on large particles will br brought rapidly to the ground, that lodged in small particles will remain suspended in the atmosphere for long periods of time. Another of the initial conditions required in the fallout model is, therefore, the distribution of activity with particle size in the stabilized cloud.

One of the features of the fission products which must be considered is the rate of decay. If the rate of decay is such that little radioactivity is left when the particles reach the ground, the effect on life will be considerably lessened. The elements decay according to an exponential law, each with its own characteristic half life. When the decay rates of all the different elements are added together, the decay of the mixture is given approximately by a power law.

$$A(t) = A(1)t^{-1.2}$$

Because of this decay it is necessary to carefully consider the time when particles reach the ground in computing the pattern of dose rate.

Figure 1 shows the distribution of activity with particle size as reconstructed from two fallout patterns observed after surface bursts. Within the range of uncertainty of measurement, this distribution appears to be valid for both small devices in Nevada and large devices in the Pacific. The limited amount of data available did not provide the means for estimating the effect of soil type on particle size distribution.

Given the stabilized cloud and the associated distribution of activity with particle size and in space, it is necessary to determine the position of the activity after it falls to the ground. The rate of fall is of primary importance here. The first analytical method which came to mind for describing rate of fall was Stoke's Law, but it was soon found by examining fallout samples that some large particles were in the range of one thousand microns diameter, and Stoke's Law does not apply to such large particles. Therefore, an aerodynamic law of fall was used which takes into account, by means of drag coefficients, inertial as well as viscous forces. Figure 2 shows the comparison of different fall rates based on experimental data. The solid curves are based on Stoke's Law for 10, 50 and 500 micron radius particles. Note that Stoke's Law is not sensitive to the density to the atmosphere; thus the variation of

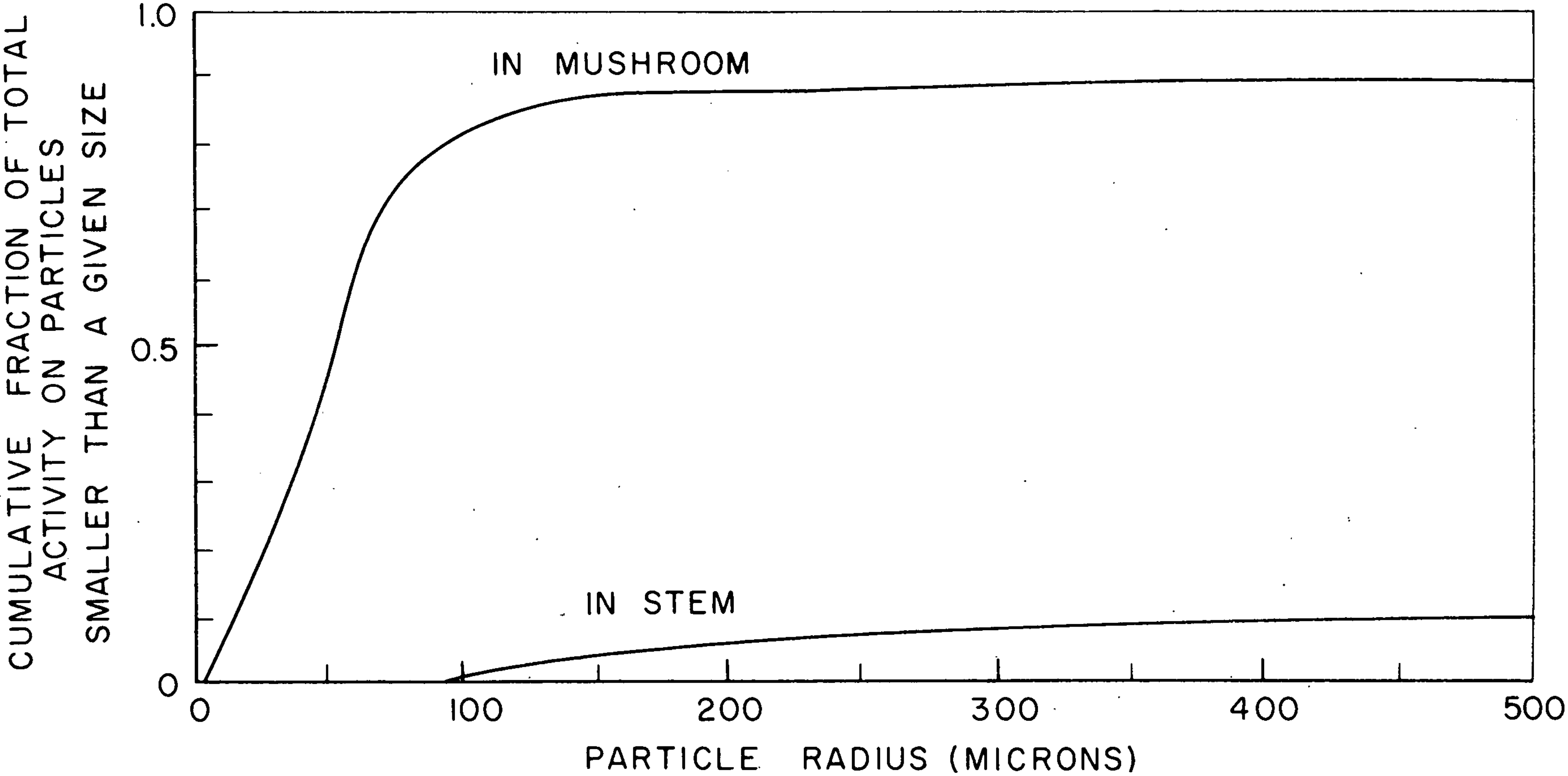

FIG. 1— RADIOACTIVITY AS A FUNCTION OF PARTICLE SIZE FOR THE STEM AND MUSHROOM OF AND ATOMIC CLOUD

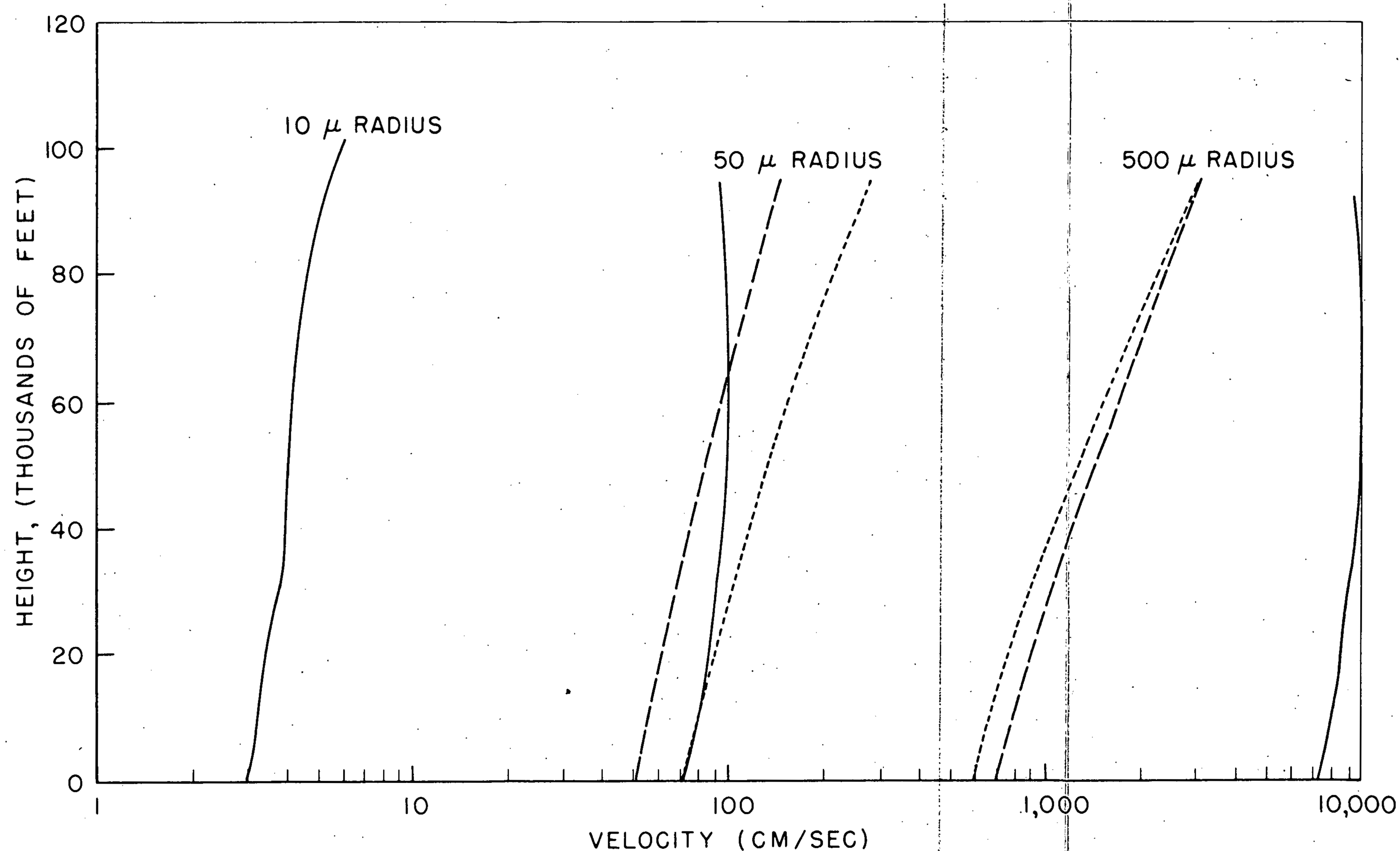

FIG. 2—THE TERMINAL VELOCITY OF PARTICLES IN THE ATMOSPHERE AS A FUNCTION OF HEIGHT FOR VARIOUS PARTICLE SIZES

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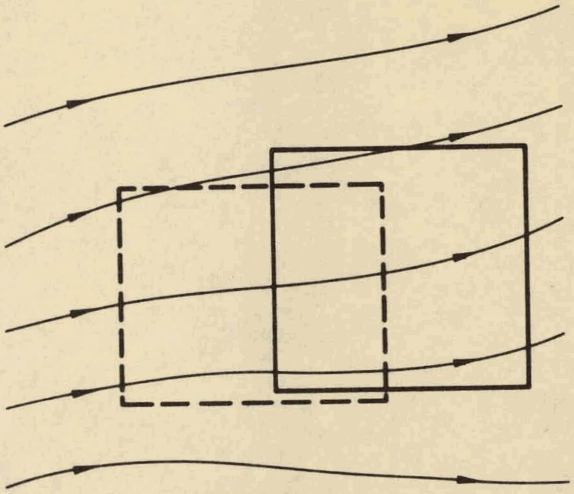
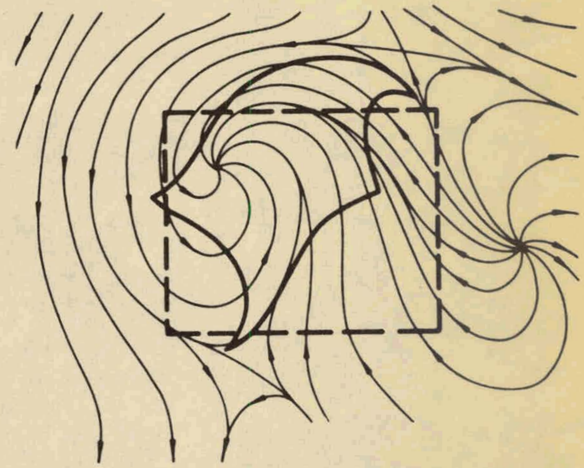
fall velocity with height is due solely to changes in viscosity. The dashed curves, representing the aerodynamic law for perfectly smooth spheres, show that the fall rate is sensitive to the density of the atmosphere and further that various particle sizes are affected quite differently. Now, the actual particles involved are by no means smooth spheres; thus, it is not quite proper, although we have done this in our calculations, to use the smooth-sphere law. The dotted curves show the fall velocities based on drag coefficients measured and reported in the Handbuch der Experimental Physic for what the experimenters term "ründlicher Korper."

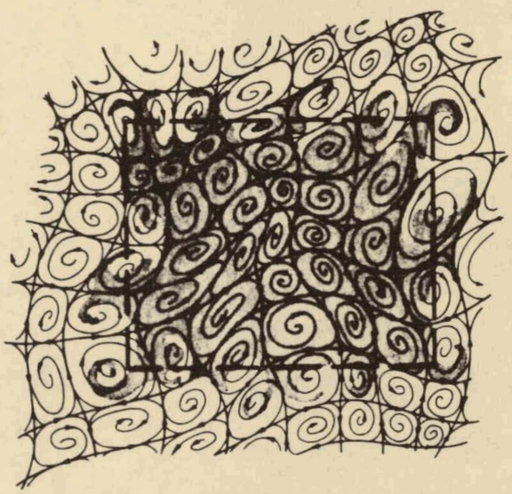
Of course, as the particles start to fall through the atmosphere, they will be carried by the wind. There are a great many wind effects which have to be taken into account. First, the whole cloud will be moved bodily by the mean wind at the cloud level. At the same time, there will be shearing effects due to the variation of the wind with altitude and turbulent effects, small scale eddies, which will tend to spread and tear the cloud. Figure 3 shows the wind effect. This picture, prepared by Dr. James Edinger of UCLA, shows cubic clouds in order to demonstrate best how the different sizes of eddies affect the clouds. Sketch (a) shows the simple translation due to the mean wind. Sketch (b) shows what would happen if there were eddies of the same dimension as the cloud, preserving it fairly well, but turning and twisting it. Sketch (c) shows that if there were eddies of an order of magnitude smaller than the cloud, little wisps of debris would be pulled out and wisps of clear air drawn in. The effect of shear is shown in sketch (d) – the cloud will be drawn into a long ribbon. If all these motions of the atmosphere are put together, sketch (e) shows what the square



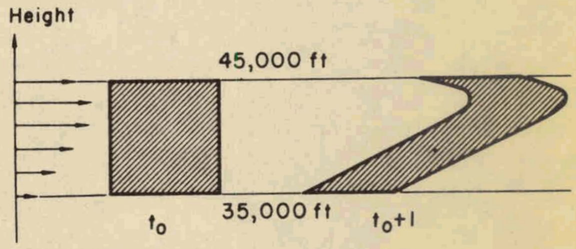
(A) Gross movement—atmospheric motions of a scale large compared with cloud dimensions



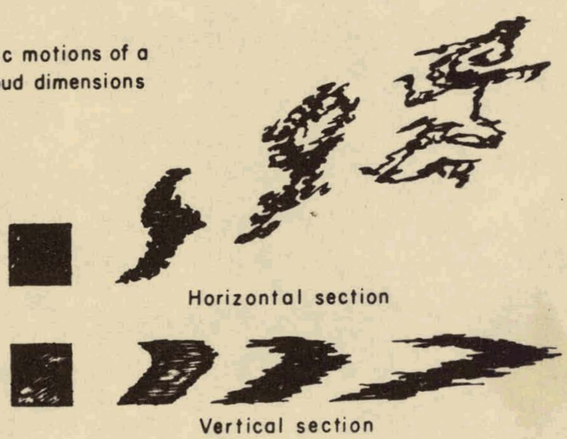
(B) Horizontal deformation—atmospheric motions of a scale comparable to cloud dimension



(C) Eddy diffusion—atmospheric motions of a scale small compared with cloud dimensions



(D) Vertical deformation —shear of horizontal wind in the vertical



(E) Total effect of gross movement, deformation, and eddy diffusion

Fig. 3

cloud would look like after a period of time under the combined influence of all these motions - - not exactly a pleasant thing to try to deal with mathematically.

It must be obvious without going into the details that there are many facets of this problem which are not well understood. The distribution of activity with size is not well known. The spatial distribution of the activity has not been measured, and the small-scale motions of the atmosphere present a statistical fluctuation which can not be predicted. However, in order to make an approximation, a simplified picture has been constructed to try to explain fallout. Figure 4 shows the cloud in model form. It rises to height H_t; H_t is a function of the amount of energy released and the ambient atmospheric conditions at the time. It is assumed to be a cylinder (the mushroom) of a certain diameter and thickness set on a stem of about 1/5 this diameter.

As far as the activity distribution within this cloud is concerned, it has been assumed that most of the activity, because of the great stability of the smoke-ring circulation, will rise into the upper part, with a small amount trailing down into the stem. It is assumed that there is a constant mixing ratio through the mushroom. This means that a count of particles per unit volume will show an exponential decrease through the mushroom. It has been assumed also that the concentration is constant throughout the stem.

As for the distribution of activity with different particle sizes, recourse to some classified results has yielded a rough idea of the activity on particles of various sizes. This is shown in Fig. 1. For fall velocities a choice of laws exists. The roundish body results reported in the literature

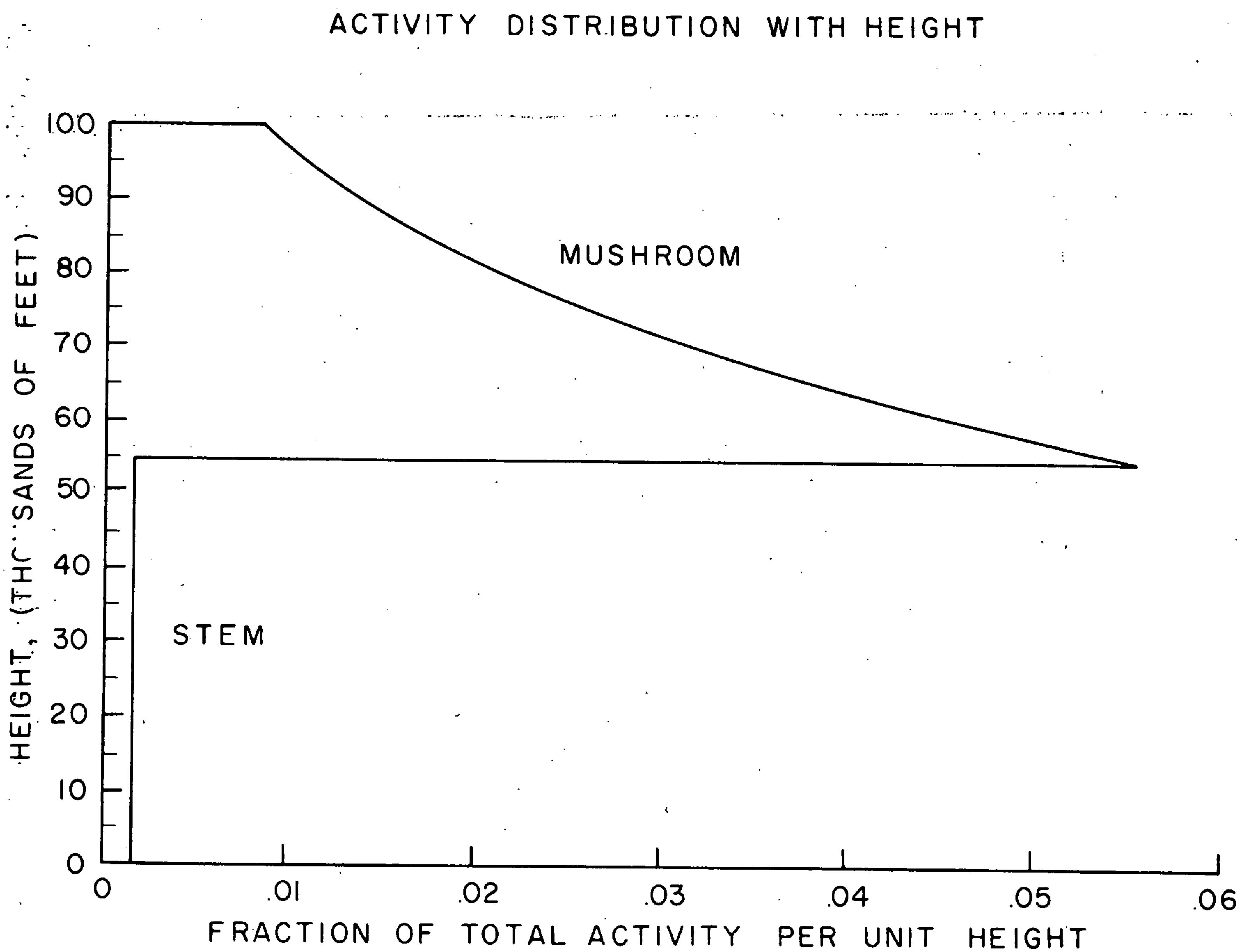


FIG. 4A—ASSUMED DISTRIBUTION OF RADIOACTIVITY IN SPACE FOLLOWING A LARGE YIELD NUCLEAR DETONATION

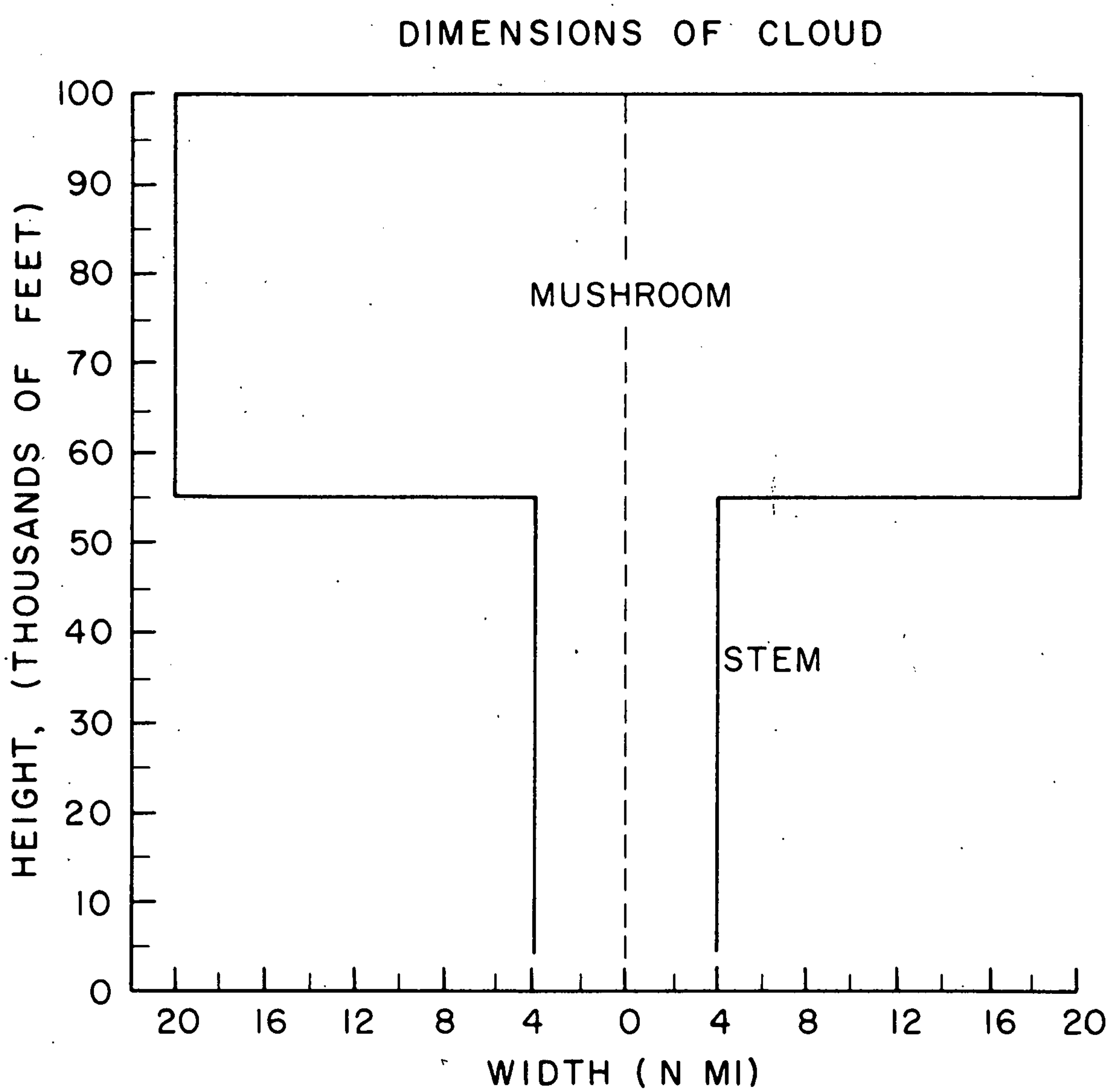


FIG. 4B — ASSUMED DISTRIBUTION OF RADIOACTIVITY IN SPACE FOLLOWING A LARGE YIELD NUCLEAR DETONATION seem most consistent with the physical picture, but to simplify the calculation, the smooth sphere results of Fig. 2 were used.

With these simplifications it is possible to make some arithmatic computations and produce a calculated fallout pattern which will serve for a comparison with observed results. In The Effects of Atomic Weapons, Appendix F, the equations for the trajectories of particles are shown. These have been modified slightly and put into the following form:

$$x = \int_0^H \left[\frac{V_{x(h)}}{W(r,h)} \right] dh$$

where x is the distance out along some coordinate, r is the radius of particle, H is the height at which it starts, W is the fall velocity and V is the wind velocity. Note that the turbulent diffusion has been ignored; it is assumed that the effect is small compared with the effects of shear and translation. We hope that diffusion will, in part, counterbalance errors in our assumption of a cylindrical cloud.

The cloud was divided into increments of height, and each slice was divided into increments of particle size. The trajectories for each of these groups of particles was then computed for the center of the cloud. This procedure has been programmed for the IBM-701, which provided for a hundred different intervals of height and a hundred different intervals of particle size. The position formulae can be applied to get the xy position of the center of any given group of particles of size r from height H; from the activity distribution, the radioactive fission product associated with these particles is known. The groups of particles are assumed to maintain a circular pattern. When all the positions on the ground are known, the contributions from all of the different (rH) groups to the radioactivity at a series of points can be summed. The pattern of radioactive fallout can be drawn from the data at these points.

In order to show the tremendous effect of the wind on the patterns of fallout, Figs. 5 and 6 are presented. These two patterns were calculated using the model just described. The parameters which are functions of bomb yield have been held constant, and the winds have been changed. Figure 5 shows the patterns at 1/2, 1, 2, and 6 hours for a moderately strong wind with little shear. Figure 6 shows the patterns for the same times but under conditions of light and variable winds. These patterns are probably extreme cases, but they were both calculated from actual wind soundings and serve to demonstrate the critical nature of the meteorological factors of the fallout problem.

The model is crude, and it would not be reasonable to expect that by assuming certain facts about the size, shape, and distribution in the cloud an accurate prediction could be made for an actual event. However, patterns so constructed are similar to those that are found after test shots. This leads us to believe that we are on the right track physically and that we will be able to improve our model and knowledge of fallout by following along this track and trying to get the necessary data to replace our assumptions with observed facts.

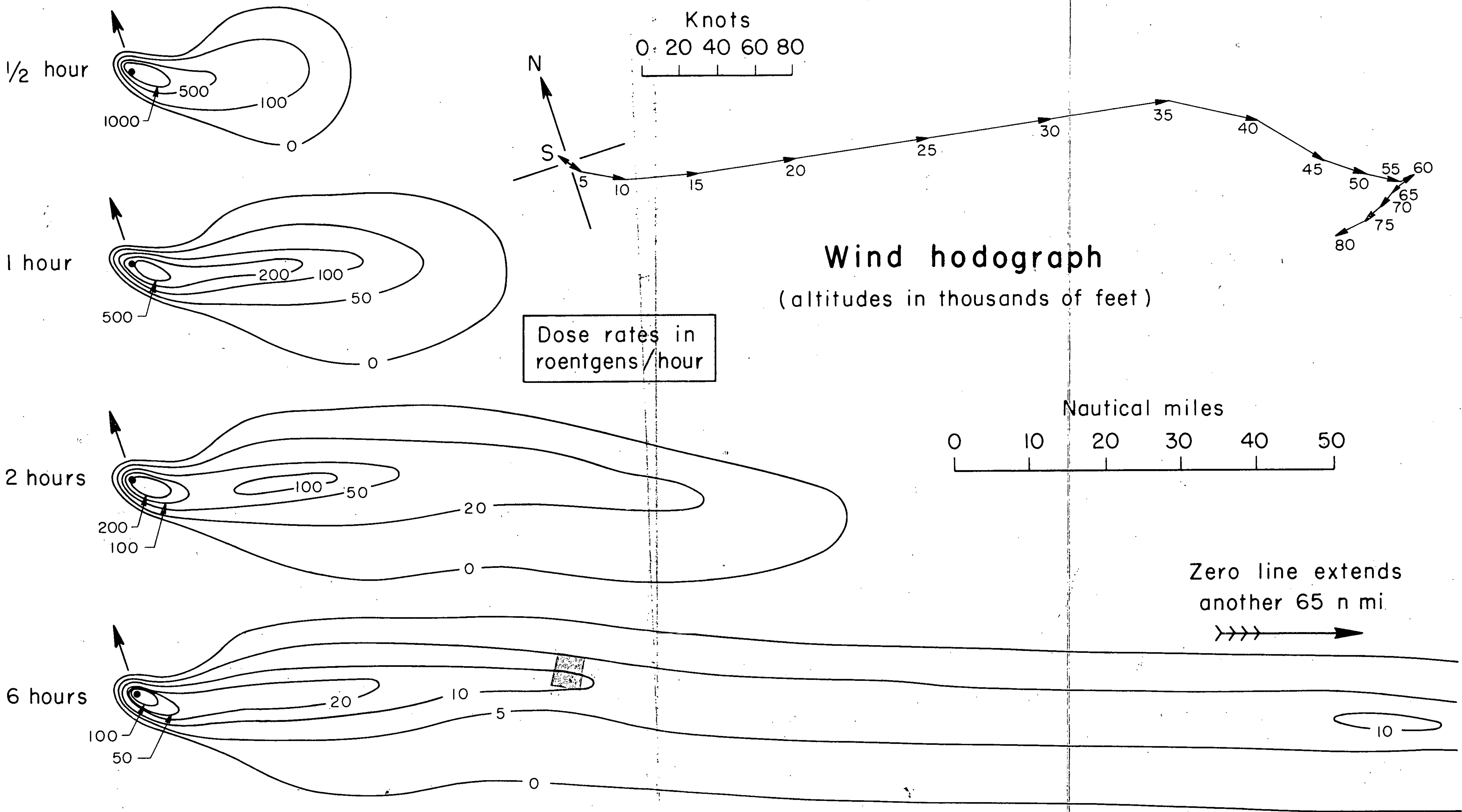

½ hour
1000
500
100
0
1 hour
500
200
100
50
0
2 hours
200
100
100
50
20
0
6 hours
100
50
20
10
5
0
10
Dose rates in roentgens/hour
N
S
Knots
0 20 40 60 80
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
Wind hodograph
(altitudes in thousands of feet)
Nautical miles
0 10 20 30 40 50
Zero line extends another 65 n mi

Fig. 5

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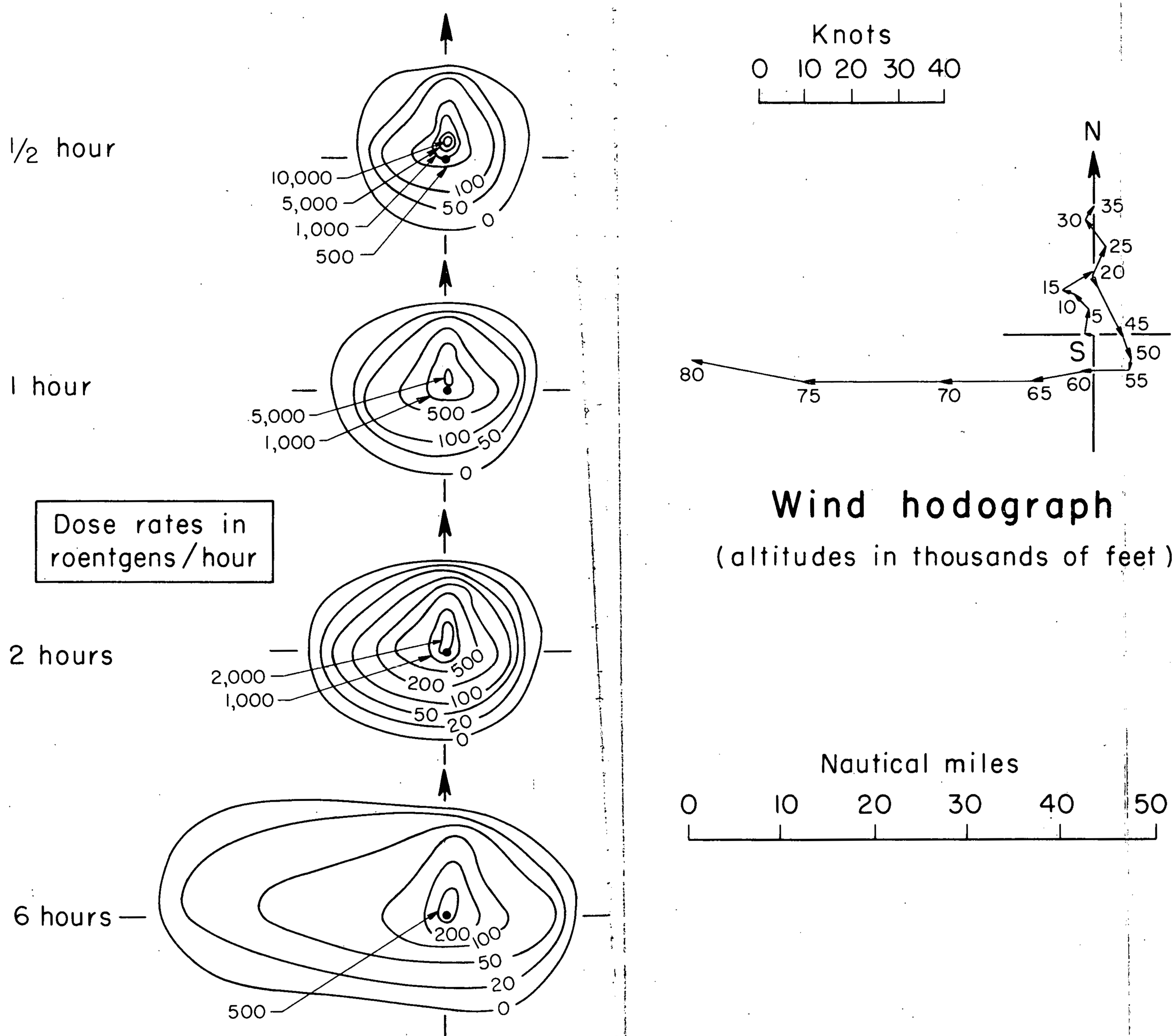

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Fig. 6